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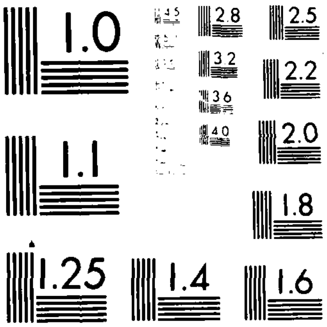
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CORRELATION IN COUPLED QUEUES AND SIMULATION OF A STOCHASTIC APPROXIMATION PROCEDURE FOR MULTIACCESS COMMUNICATIONS

WALTER E. BRADY

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In this report, two seperate topics in queing theory are discussed. In the first half of the thesis, methods are presented that are used to show that quantitative evidence of correlation exists in intuitively correlated coupled systems. The second half of this thesos concerns the use of simulations to verify that predictions are valid for specific random access control strateigies. These control strategies include cases where the number of stations is allowed to slowly vary.

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CORRELATION IN COUPLED QUEUES AND SIMULATION OF A STOCHASTIC APPROXIMATION PROCEDURE FOR MULTIACCESS COMMUNICATIONS

BY

WALTER EDWARD BRADY

B.S., University of Illinois, 1983

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1985

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Dedication

To My Father

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Chapter I

Evidence of Correlation in Coupled Systems

1.A. Introduction

In this thesis, two separate topics in queueing theory are discussed. The first topic concerns the quantitative analysis necessary to support intuitive claims that correlation exists in a system. The second topic investigates the simulation of certain dynamic strategies used to control the transmissions in a time-slotted packet radio multiaccess broadcast channel under the condition that the number of users, N, is allowed to vary slowly.

The dependence due to access protocols intuitively introduces correlation into a system of nodes in a packet radio network. However, the analysis necessary to support this claim has been found to be extremely complex and tedious. One of the largest problems is simply trying to produce a closed form expression for the generating function of a coupled network. This thesis shows that there are methods which produce results that support the claim that correlation does indeed exist in "intuitively" correlated systems.

In [6] and later in [7], work is done to determine average waiting times and average queue lengths for two node systems. A method discussed there is used to determine conditional and unconditional expected queue lengths for symmetric systems. In [1], a method is introduced to determine a closed form expression of a generating function for a system of coupled processors by reducing the problem to a Riemann-Hilbert problem. This paper gives the only method known to solve for the generating function of a coupled system. In [8], the distribution of the waiting time of a customer is determined for a shared server system. A general background for problems exhibited by coupled processors is found in [6], and a general background for packet radio communications where much of this work is applicable is found in [5].

In this chapter, the discussion of correlation begins with describing the goals of this work. Models are presented and an efficient method is used to determine the generating functions of the models. The generating functions are then analyzed to produce results consistent with the goals already presented. The results found concern the ordering of the conditional and the unconditional moments of the systems, the ordering of the poles of the generating functions given for some systems, and the phenomenon that hinders the search for moments of higher order. An ALOHA model is then presented along with a set of goals. In this model, a conjecture is supported through simulation; however, a valid proof of this conjecture is an open problem. This is followed by the second chapter of this thesis, simulation of dynamic strategies used to control transmission probabilities.

1.A.1. Goals and Models

The goal of this chapter simply stated is to produce quantitative evidence as to the existence of correlation in a system that exhibits correlation intuitively. At the basis of this work is the idea of intuitively correlated versus quantitatively correlated. Although one system may seem to be more intuitively correlated than another, a measure is needed to make such a claim precise. With quantitative evidence, such claims are given concrete meaning.

Two such quantitative results concern the generating function determined by the system. The first is a relationship between the conditional and unconditional moments of the system. If the unconditional expected number of packets in a station, the queue-server combination defined later for the models in this paper, is greater or less than the conditional expected number of packets in a station, given that the other station is empty, correlation exists between the two stations.

The second evidence used to imply correlation concerns the poles of the generating function of the random variable determined by the total number of packets in the two stations. The basis for the argument that poles of the generating function are a possible indicator of correlation lies in the relationship that exists between the pole placement and how quickly the tail of the distribution function goes to zero. It is seen that in an uncorrelated system, the tail goes to zero faster than in a correlated system. Intuitive evidence that this is true is found in the argument that in a correlated system it is expected that if one station has a large backlog, then the other station also has a large backlog. This results in the random variable determined by the sum of the packets in each station having a distribution function with a heavier tail than the tail of the distribution function of an uncorrelated system. A heavier tail simply means that the tail goes to zero more slowly than a comparable tail. This implies that an ordering of the poles of a system's generating functions produces evidence of correlation.

Further evidence is sought involving higher order moments of the generating functions. For example, if the first joint moment of the system were greater than the product of the first moments of each station, a correlation coefficient could be determined and used as evidence of correlation. This evidence, however, is unattainable using the methods presented. The problem lies in the fact that there are more unknowns than equations available to be solved for. In fact, for one method presented, there are exactly twice as many unknowns as there are equations to solve them.

An emphasis is put on evidence of correlation in this chapter; however, the discussion of higher moments is felt to be necessary since it is directly linked to the methods used to gain results for correlation.

1.A.2. Models used in the Analysis

Three different systems are discussed and defined here. The first is a system which will be called the splitting system. It has customers arriving which obey a Poisson arrival process. Upon arriving, the customers are split into two equal packets, and each packet is then queued and served separately by two stations. The arrival rate is λ and the service times are exponentially distributed with rate μ. If each station, the queue-server combination, is looked at separately, the stations look like simple M/M/1 queues with arrival rate λ and departure rate μ. This system exhibits intuitive correlation since when a customer arrives, both stations receive a packet. Therefore, if a station is empty, it is expected that the other station should have a smaller backlog in its queue than if the unconditional backlog is looked at. Hence, this is an example of a correlated system.

The next system discussed is similar to the splitting system in that the stations again act independently upon receiving a packet. The difference between the splitting system and a system which will be called the independent system is in the arrival process. In the independent system, each station has a separate independent Poisson arrival process with rate λ. This system is equivalent to two independent M/M/1 queues. The results from this uncorrelated network are compared to the same results for a correlated network.

The final system has a separate independent Poisson arrival process for each station as did the independent system. This system, which shall be called the shared-server system, uses two queues and a single server. The server has exponentially distributed service times; however, the rate of departures is 2μ. The server alternates between serving the queues as long as one of the queues is not empty. This is analogous to two separate servers each with rate μ for each of the customer types, where the type of customer is determined by the queue at which it arrived, if both customer types are present to be be served. However, if one of the queues becomes empty and hence one of the customer types is no longer present in the system, the single server will put its entire effort to serving only the type of customer still present in the system with rate 2μ. Correlation exists intuitively in this system since if only one customer type is present in the system, the server will work with rate 2μ rather than with rate μ to lower the backlog in the nonempty queue.

The shared-server system, although it is an intuitively correlated system, is used only as an example in the subsection on determining the generating function. It may also be analyzed using methods similar to those presented in this paper to determine evidence of correlation. However, the first two systems presented, the splitting system and the independent system, supply ample proof that the methods used to provide evidence of correlation are sufficient in the sense that the goals set forth are satisfied.

1.A.3. Deriving the Generating Functions

This section uses an efficient method to derive the generating functions of our systems. The state of the systems is defined to be the number of customers, or packets, in station i, where i = 1,2 at time, t, in steady state. Consider the steady-state joint generating function of the number of customers in each of the stations:

$$G(x,y) = \lim_{t\to\infty} E[x^{n_1(t)} y^{n_2(t)}]$$

where $n_1(t)$ and $n_2(t)$ are the number of customers in stations 1 and 2, respectively, and the Markov chain $[n_1(t), n_2(t)]$ is ergodic. The determination of this generating function is the goal of this subsection.

Letting

$$\pi(i,j) = \text{Prob}\ (n_1(t) = i,\ n_2(t) = j)\ ,$$

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \pi(i,j) = 1,$$

and

$$G(x,y) = E[x^{n_1(t)} y^{n_2(t)}] = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \pi(i,j) x^i y^j\ , \qquad (1.1)$$

the conditions for ergodicity are that

$$\pi(0,0) = G(0,0) > 0\ ,\ \text{and}$$

$$0 = \pi Q$$

where $\pi(i,j)$ is the joint steady-state density function and Q is the transition rate matrix that corresponds to the system under study. These conditions are necessary for an ergodic generating function to exist.

The transition rate matrix may in some cases be described more simply by a transition rate diagram (see Fig. 1). This is true for the cases given by the systems discussed in the preceding section. For example, the transition rate diagram given in Fig. 1.1 is for the splitting system. This diagram represents customers arriving with rate λ by the diagonal transitions and customers departing with rate μ by the horizonal and vertical transitions. Each point on this diagram represents a possible state in which the process may be found.

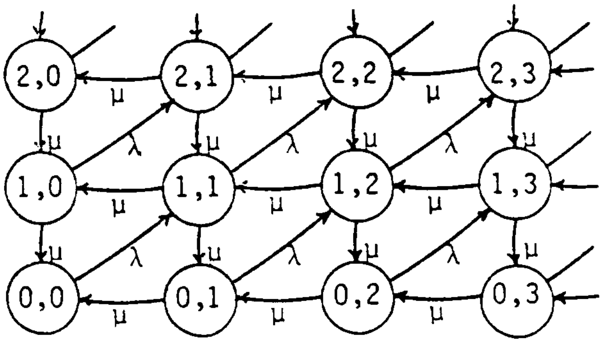

Fig. 1 Transition Diagram for the Splitting System

These points on the diagram fall into four categories, points where $i \geqslant 1, j \geqslant 1$, points where $i \geqslant 1, j=0$, points where $j \geqslant 1, i=0$, and the point $i=0, j=0$. These categories also represent four different parts of the generating function given by equation (1.1). These parts are

$[G(x,y) - G(x,0) - G(0,y) + G(0,0)]$ for $i \geqslant 1, j \geqslant 1$,

$[G(x,0) - G(0,0)]$ for $i \geqslant 1, j=0$,

$[G(0,y) - G(0,0)]$ for $i=0, j \geqslant 1$, and

$G(0,0)$ for $i=0, j=0$.

Each of these parts, for the splitting system, has a different arrival and departure process assigned to it. For example, the point $i=0, j=0$ may have only arrivals whereas the point $i \geqslant 1, j \geqslant 1$ may have both arrivals and departures.

Under the conditions supplied by the transition diagram, the allowable transitions for the four categories of points are now incorporated to determine the generating function form of the equilibrium relationship. For this case the equilibrium relationship is $0 = \pi Q$. For the splitting system, the allowable transition determined by an arrival introduces the term $\lambda(xy-1)$ since an arrival is seen as an arrival to both stations. A departure from station 1 is represented by $\mu(x^{-1}-1)$, and a departure from station 2 is represented by the similar result $\mu(y^{-1}-1)$. Hence, the generating function form of the equilibrium relationship for the points of the type $i \geqslant 1, j \geqslant 1$ is simply

$$[G(x,y) - G(x,0) - G(0,y) + G(0,0)][\lambda(xy-1) + \mu(x^{-1}-1) + \mu(y^{-1}-1)].$$

Using this technique, the entire generating function form of the equilibrium relationship for the splitting system is given by

$$\begin{aligned}0 = {} & [G(x,y) - G(x,0) - G(0,y) + G(0,0)][\lambda(xy-1) + \mu(x^{-1}-1) + \mu(y^{-1}-1)] \\ & + [G(x,0) - G(0,0)][\lambda(xy-1) + \mu(x^{-1}-1)] \\ & + [G(0,y) - G(0,0)][\lambda(xy-1) + \mu(y^{-1}-1)] \\ & + G(0,0)[\lambda(xy-1)] .\end{aligned}$$

This method is easily applied to the independent system where the transitions allowed are of a different form than those of the splitting system. A customer arriving at this system introduces terms of the form $\lambda(x-1)$ and $\lambda(y-1)$ depending on whether the customer arrives at station 1 or station 2, respectively. The generating function form of the equilibrium relationship then takes on the result

$$\begin{aligned} 0 = {} & [G(x,y) - G(x,0) - G(0,y) + G(0,0)][\lambda(x-1) + \lambda(y-1) + \mu(x^{-1}-1) + \mu(y^{-1}-1)] \\ & + [G(x,0) - G(0,0)][\lambda(x-1) + \lambda(y-1) + \mu(x^{-1}-1)] \\ & + [G(0,y) - G(0,0)][\lambda(x-1) + \lambda(y-1) + \mu(y^{-1}-1)] \\ & + G(0,0)[\lambda(x-1) + \lambda(y-1)] \, . \end{aligned}$$

The shared-server system also has different transitions allowed than the forementioned systems. For the points $i \geqslant 1, j \geqslant 1$ and the point $i=0, j=0$, the transitions allowed are the same as in the independent system, but for points on the $i=0, j=0$ axes the departure rate is increased to 2μ. This introduces a term of the form $2\mu(x^{-1}-1)$ or $2\mu(y^{-1}-1)$ into the generating function form of the equilibrium relationship. This result,

$$\begin{aligned} 0 = {} & [G(x,y) - G(x,0) - G(0,y) + G\ (0,0)][\lambda(x-1) + \lambda(y-1) + \mu(x^{-1}-1 + \mu(y^{-1}-1)] \\ & + [G(x,0) - G(0,0)][2\mu(x^{-1}-1) + \lambda(x-1) + \lambda(y-1)] \\ & + [G(0,y) - G(0,0)][2\mu(y^{-1}-1) + \lambda(y-1) + \lambda(x-1)] \\ & + [G(0,0)[\lambda(x-1) + \lambda(y-1)] \end{aligned}$$

is consistent with the previous results.

This method can also be applied to discrete time processes which have an equilibrium form of $\pi = \pi P$, where π is previously defined and P is the transition probability matrix. However, the only example of this type of system is given in section 1.C. The results for the splitting system and the independent system may be rewritten as

$$G^S(x,y) = \frac{G^S(x,0)(y^{-1}-1) + G^S(0,y)(x^{-1}-1)}{(x^{-1}-1) + (y^{-1}-1) + \rho(xy-1)} \qquad (1.2)$$

and

$$G^I(x,y) = \frac{G^I(x,0)(y^{-1}-1) + G^I(0,y)(x^{-1}-1)}{(x^{-1}-1) + (y^{-1}-1) + \rho(x-1) + \rho(y-1)} \tag{1.3}$$

where $\rho = \lambda/\mu$ and $G^S(x,y)$ and $G^I(x,y)$ are the notations used to denote the generating function of the splitting system and the independent system, respectively. These two results are now used to show that the goals set at the beginning of this chapter may be met.

1.B. Indicators of Correlation

In this section, the ordering of the conditional and of the unconditional moments of first order of the systems is investigated. Following this discussion, the other evidence of correlation is then presented using an argument which depends on the ordering of the poles of a generating function. Higher moments of the systems are investigated as the final topic of this section.

1.B.1. Ordering of the First Order Conditional and Unconditional Moments

Both the independent and splitting systems are analyzed to determine the ordering of their higher order moments. Equations (1.2) and (1.3) are multiplied by xy in the numerator and in the denominator to simplify the analysis. Some simple properties of the generating functions are now stated which will be used later in the analysis. Since the systems studied are symmetric,

Property 1 $G(x,y) = G(y,x)$, and

Property 2 $G_{1^k 2^m}(x,y) = G_{1^m 2^k}(y,x)$, where

$$G_{1^k 2^m}(x,y) = \frac{\partial^k}{\partial x^k} \frac{\partial^m}{\partial y^m} G(x,y)$$

are both properties of the generating function. Another important property is:

Property 3 $$\lim_{x \to 1} G_{1^k}(x,1) = \lim_{x \to 1} G_{1^k}(x,x) \quad .$$

This is a consequence of the analyticity of the generating function. The final property presented follows directly from calculus:

Property 4 $$\frac{d}{dx}[G(x,x)] = G_1(x,x) + G_2(x,x) \quad .$$

Using these properties, the conditional and unconditional first order moments are now determined for the systems under investigation. Substituting in y=1 for the altered (1.2) as described above, and using property 1, the result that

$$G^S(x,1) = \frac{G^S(0,1)}{1-x\rho} \tag{1.4}$$

is attained. Similarly, for the independent system, the result that

$$G^I(x,1) = \frac{G^I(0,1)}{1-x\rho} \tag{1.5}$$

is found. Using the common normalizing condition, G(1,1) = 1, the result that

$$G^S(0,1) = G^I(0,1) = (1-\rho)$$

provides the probability of one of the stations being empty. Expanding the results from (1.4) and (1.5), the equation that

$$\frac{d^k}{dx^k} G^I(x,1) = \frac{d^k}{dx^k} G^S(x,1) = \frac{k\rho^k(1-\rho)}{(1-\rho x)^{k+1}} \qquad (1.6)$$

is produced. When k is allowed to equal 1 and x=1, the unconditional expected number in a station is given by (1.6).

To derive the conditional expected number in a station given that the other station is empty, the derivative of G(x,y) given by (1.2) and (1.3) for each system is taken with respect to x (or y). This result is followed by setting y=x which produces the following for the splitting system:

$$G_1{}^S(x,x) = \frac{G_I^S(x,0)}{(2-x\rho(1+x))} + \frac{G^S(x,0)\rho(2x+1)}{(2-x\rho(1+x))^2} \qquad (1.7)$$

and for the independent system

$$G_1{}^I(x,x) = \frac{G_1{}^I(x,0)}{(2-x\rho)} + \frac{2\rho G^I(x,0)}{(2-x\rho)^2} \quad . \qquad (1.8)$$

Allowing x→1, $G_1(1,0)$, which is equal to $G_2(0,1)$ by property 2, is attained for both systems:

$G_1{}^S(1,0) = \rho/2$, and

$G_1{}^I(1,0) = \rho$.

These results are used to gain the expected number in one station given that the other is empty, since this value is equal to $G_1(1,0)/G(1,0)$. Hence, the conditional first order moment for one station, given that the other station is empty, is $\rho/2(i-\rho)$ for the splitting system and $\rho/(1-\rho)$. for the independent system. These results provide the first quantitative evidence that the splitting system is correlated and that the independent system is uncorrelated. Hence, the conditional moment is less than the unconditional moment in the splitting system which corresponds to the intuitive claim that if one station is empty, the other station tends to have a smaller backlog than when the backlog is not conditioned upon having one station empty. In the independent system, both moments are equal, implying that conditioning upon an empty station does not affect the backlog in the other station, and that the system is uncorrelated.

This technique provides only one piece of evidence as to the existence of correlation. Further insight into the techniques available to produce evidence of correlation is necessary and given in the next subsection.

1.B.2. Ordering of Poles

This subsection discusses the ordering of the poles of the generating function for the random variable defined as the sum of the number of customers in stations 1 and 2 for the splitting system and for the independent system. A third system is also defined to provide another pole to compare to the poles already found.

This system is an intuitively totally correlated system defined by two stations using an arrival process similar to that of the splitting system. However, the customers must also depart at the same time unlike either the splitting or the independent systems. This simply says that if a packet arrives at the network, it appears as a customer arriving at each station. If a customer departs from one station, there is always a departure from the other station. The arrival process is Poisson with rate λ, and the departure process has an exponentially distributed service time with rate μ. Using the technique described in subsection 1.A.3., the joint steady-state generating function is determined to satisfy

$$G^{TC}(x,x) = \frac{1-\rho}{1-\rho x^2} \quad \text{where} \quad \rho = \lambda/\mu \tag{1.9}$$

for $y=x$. This is sufficient since the generating function of the random variable defined by the sum of the number of customers in each station is equivalent to setting $y=x$ in the joint steady-state generating function. Another way of looking at this system is to view it as just a M/M/1 queue which allows only two customers to arrive or to depart instead of just one customer arriving or departing. This system can be interpreted as totally correlated since each station contains the same number of packets.

The tie between the poles of the generating function of the random variable, which is defined by the sum of the number of customers in each station denoted by Z and correlation, lies in the relationship between the poles and the tail of this random variable. It is known that for station i, i equalling 1 or 2, each of the three systems discussed has the same marginal distribution. Therefore, a heavier tail for the random variable given by the sum of the number of of packets at each station indicates that more correlation exists in that system. So, by determining if the tail for one system is heavier than that of another, evidence is obtained supporting the claim that more correlation exists in one system than in another. One way to measure how heavy a tail is is to look at the least magnitude pole of the generating function of Z. For example, for the geometric distribution with parameter β, the pole of the generating function, given by $1/\beta$, is inversely proportional to the magnitude of β. This results in a heavier tail for a larger β, or equivalently for a smaller least magnitude pole. In general, roughly speaking, if for one system, the pole of least magnitude is found to be less than that for another system, the first system has a tail that goes to zero faster than the second system's tail.

Using equations (1.2) and (1.3), where y is set equal to x, and (1.9), the poles of the generating function of the random variable, Z, are found to be:

$$x^{I} = 1/\rho$$

$$x^{TC} = 1/\sqrt{\rho} \quad , \quad \text{and}$$

$$x^{S} = \frac{(\rho^{2}+8\rho) - \rho}{2\rho}$$

where x^{j}, $j=I$, TC, or S, corresponds to the poles of the independent, totally correlated, and splitting systems, respectively. The poles of totally correlated and independent systems are easily determined. However, the poles of the splitting system are also affected by the location of any possible poles of the unknown $G^{S}(x,0)$. And, for x real,

$$G^S(0,x) \leqslant G^S(1,x)$$

and the term $G^S(1,x)$ is known to have a first pole at $1/\rho$, so, since $G^S(0,x)$ can not go to infinity prior to $G^S(1,x)$, $G^S(0,x) \leqslant +\infty$ for $0 \leqslant x \leqslant 1$. So, any pole of $G^S(0,x)$ does not interfere with the placement of the first major pole for the splitting system, and the ordering of the poles follows as

$$x^{TC} < x^S < x^I \qquad \text{for} \qquad \rho \varepsilon (0,1) \ .$$

So, by using the tail argument, this ordering implies that the splitting system is more correlated than the independent system, and less correlated than the totally correlated system.

This technique provides the second piece of evidence used to show the existence of correlation in a system. Further evidence is sought in the next section, but no definitive results are produced.

1.B.3. Higher Order Moments

In this subsection, relationships between the higher moments are given and a general method to determine these relationships is also discussed. Although the ultimate goal is to solve for these moments explicitly, this goal is unattainable using the methods given here at this time. However, it is of interest that using a method presented here, only one piece of information is gained for every two iterations of the method. Two methods are discussed in this subsection. The first is an extension of the work done in subsection 1.B.1. while the second uses the analyticity of the denominator of the generating function around the point x=1,y=1 on the curve defined by the zeroes of the denominator. When the sets of equations found using these two methods are combined, there are still not enough equations to solve for any of the higher order unknowns. Both methods are applied only to the splitting system here. However, other systems may also be analyzed using these methods.

The first method discussed is a direct extension of the analysis done in subsection 1.B.1. The same properties hold, and the results found there are necessary here to continue this analysis. Using the result already found for $G_1^S(x,y)$, the derivative with respect to y is taken. Again, set y=x and allow $x \to 1$. The result for $G_{12}^S(1,1)$ is the following:

$$G_{12}^S(1,1) = \frac{G^S_{1^2}(1,0)}{2(1-\rho)} + \frac{(\rho^2+\rho)}{2(1-\rho)^2} \quad . \tag{1.10}$$

This equation has two unknowns, so it is not solvable for either unknown. Further derivates only increase the number of unknowns by two for each new equation, namely, two new joint moments of the form

$$G^S_{1^k 2^m}(1,1) \quad \text{and} \quad G^S_{1^{k+m}}(1,0) \; .$$

This does produce a set of (k+m-1) equations in 2(k+m-1) unknowns which, upon the introduction of (k+m-1), more independent equations in the same unknowns are solvable.

The second method to determine higher order moment equations revolves upon analyzing the zeroes of the numerator of the generating function in a neighborhood around $x=1,y=1$ on the curve determined by the zeroes of the denominator of the generating function. The splitting system is used as an example for this method, and equation (1.2) is altered by multiplying the numerator and the denominator by xy to produce

$$G^S(x,y) = \frac{H(x)x(1-y) + H(y)y(1-x)}{x(1-y) + y(1-x) + \rho xy(xy-1)} \tag{1.11}$$

where $H(x) = G^S(x,0)$ and $H(y) = G^S(0,y)$. This is a much simpler form of the generating function, and it is much easier to analyze in this form. Now the curve defined by setting the denominator to zero is

$$y(1-x) + x(1-y) + \rho xy(xy-1) = 0 \quad . \tag{1.12}$$

The point $x=1,y=1$ is a solution to this equation, hence it lies on the curve determined by equation (1.12). Substituting $x=1,y=1$ into the numerator of equation (1.11), it is found that this point is also a solution to the equation,

$$xH(x)(1-y) + yH(y)(1-x) = 0 \; . \tag{1.13}$$

Therefore, the point $x=1,y=1$ results in an indeterminate form for $G^S(1,1)$. It is known that $G^S(x,y)$ is a generating function, so it is analytic in a ball about the origin which contains the point $x=1,y=1$.

Using this knowledge, a solution is found for equation (1.13) for a function $y = \phi(x)$. This result,

$$H(\phi(x)) = \frac{\phi(x)^{-1}-1}{(1-1/x)} H(x) \qquad (1.14)$$

and the result that is found when equation (1.12) is solved for $y=\phi(x)$,

$$\phi(x) = -\frac{(1-2x-\rho x) - ((1-2x-\rho x)^2 - 4\rho x^3)}{2\rho x^2} \qquad (1.15)$$

produces a foundation for this method. By solving for $\phi(x)$ in equation (1.15) and then substituting this result into equation (1.14), it appears it is possible to solve for H(1) and for all the derivatives of H(x) with respect to x evaluated at H(1). Before proceeding with this method, a simpler format is presented to simplify the necessary calculations.

Using the generating function defined as,

$$G^S(x,y) = \frac{N(x,y)}{D(x,y)}$$

where $N(x,y)$ is equivalent to equation (1.13) and $D(x,y)$ is equivalent to equation (1.12) and using the fact that $G^S(x,y)$ is analytic, so when $D(x,y)=0$, the equation, $N(x,y)=0$, is also true, the method produces results for $N(x,y)$. This is possible since using the $\phi(x)$ defined in equation (1.15) and using the result that,

$$N(x,\phi(x)) = 0 \quad \text{and} \quad D(x,\phi(x)) = 0$$

as well as the result that all the derivatives of $N(x,\phi(x))$ and of $D(x,\phi(x))$ are equal to zero, $N(x,\phi(x))$ and all the derivatives of $N(x,\phi(x))$ can be obtained. Using the notations,

$$\frac{\partial^k}{\partial x^k}\frac{\partial^m}{\partial y^m} N(x,y) = N_{x^k y^m} \quad ,$$

$$\frac{\partial^k}{\partial x^k}\frac{\partial^m}{\partial y^m} D(x,y) = D_{x^k y^m} \quad , \text{ and}$$

$$\frac{\partial^k}{\partial x^k} \phi(x) = \phi(x)^{\overset{k \text{ times}}{\prime\prime\prime\prime\prime\prime\prime\prime}} \qquad = \phi'^k(x)$$

it is possible to produce tables of solutions for these values. For example, Table (1.1) is a list of all the possible $D_{x^k y^m}$ values for the splitting system with x=1,y=1. The $\phi'^k(1)$ values are determined by taking derivatives of $D(x,\phi(x))$ and by solving for $\phi'^k(x)$ with x=1. For example, taking the first derivative of $D(x,\phi(x))$ results in

$$D_x + \phi'(x)D_y = 0 \quad .$$

Table (1.1) Table of $D_{x^k y^m}$ values for the point x,y=1 of the splitting system

$D(1,1) = 1$

$D_x(1,1) = D_y(1,1) = \phi - 1$

$D_{x^2}(1,1) = D_{y^2}(1,1) = 2\phi$

$D_{xy}(1,1) = 3\phi - 2$

$D_{yx^2}(1,1) = D_{yx^2}(1,1) = 4\phi$

$D_{x^2y^2}(1,1) = 4\phi$

$D_{x^k y^m} = 0$ for $(m=0,\ k>2)$, $(k=0,\ m>2)$, $(m=1,\ k>2)$, $(k=1,\ m>2)$, and $(m>2,\ k>2)$

Solving for $\phi'(1)$, the result is $\phi'(1)=-1$. Continuing this procedure, a table of $\phi'^k(1)$ is computed and compiled in Table (1.2) for $k \varepsilon \{0,1,2,3\}$.

Table (1.2) Table of $\phi'^k(1)$ values for $k \varepsilon \{0,1,2,3\}$

$$\phi(1) = 1$$

$$\phi'(1) = 1$$

$$\phi'^2(1) = \frac{4 - 2\rho}{(1-\rho)}$$

$$\phi'^3(1) = \frac{G(\rho-2)(2-\rho)}{(1-\rho)^2}$$

A table for $N_{x^k y^m}$ where $x=1, y=1$ is presented in Table (1.3).

Table(1.3) Table of $N_{x^k y^m}(1,1)$ for the splitting system

$$N_x(1,1) = N_y(1,1) = -H(1)$$

$$N_{x^2}(1,1) = N_{y^2}(1,1) = 0$$

$$N_{xy}(1,1) = -2(H(1) + H'(1))$$

$$N_{xy^2}(1,1) = N_{yx^2}(1,1) = -H'(1) - H'^2(1)$$

$$N_{x^k y^m}(1,1) = 0 \qquad \text{for } k,j \geqslant 2$$

$$N_{x^k y}(1,1) = N_{y^k x}(1,1) = -kH'^{k-1}(1) - H'^k(1)$$

Using a similar method as was used to solve for $\phi(x)$ and for the derivatives of $\phi(x)$ by the solution to the equations using the $D(x,y)$

terms, it is possible to produce an equation in H(x) and in the derivatives of H(x) by using the N(x,y) terms in place of the D(x,y) terms. For example, the first derivative of N(x,y), where $y = \phi(x)$, produces,

$$N_x + N_y\,\phi'(1) = 0 \quad . \tag{1.16}$$

Using the values found in Tables (1.2) and (1.3), it is seen that $N_x = -N_y$ which is just an identity. Consequently, from this point on all the N(x,y) terms are evaluated at x=1, y=1, so the (1,1) has been dropped. The next derivative of equation (1.16) produces

$$N_{xx} + 2\,N_{xy}\,\phi'(1) + N_{yy}(\phi'(1))^2 + N_y\,\phi'^{2}(1) = 0 \ . \tag{1.17}$$

Solving this for H'(1) using Tables (1.2) and (1.3) results in

$$H'(1) = \rho/2 \quad .$$

This is the same result found in subsection 1.B.1. for $G_1(1,0)$. The next derivative of equation (1.17) produces another identity as was the case for equation (1.16). The next derivative following this one results in the equation

$$\begin{aligned} &N_{x^4} + 4\,N_{yx^3}\,\phi'(1) + 6\,N_{x^2y^2}(\phi'(1))^2 + 6\,N_{yx^2}\,\phi'^{2}(1) + 4\,N_{xy^3}(\phi'(1))^3 \\ &+ 12\,N_{xy}(\phi'(1))(\phi'^{2}(1)) + 3\,N_{y^2}(\phi'^{2}(1))^2 + 4\,N_{x^2}(\phi'(1))(\phi'^{3}(1)) \\ &+ N_{y^4}(\phi'(1))^4 + 6\,N_{y^3}(\phi'(1))^2(\phi'^{2}(1)) + 4\,N_{yx}\,\phi'^{3}(1) \\ &+ N_y\,\phi'^{4}(1) = 0 \end{aligned} \tag{1.18}$$

which simplifies to

$$4\,N_{x^3y}(\phi'(1)) + 6\,N_{x^2y}\,\phi'^2(1) + 4\,N_{xy^3}(\phi'(1))^3$$

$$+\,12\,N_{xy^2}(\phi'(1))(\phi'^2(1)) + 4\,N_{xy}\,\phi'^3(1) + N_y\,\phi'^4(1) = 0 \qquad (1.19)$$

upon the substitution of the terms known to be zero from Table (1.3). Now, using

$$\phi'^4(1) = \frac{24(8-11\rho + 6\rho^2 - \rho^3)}{(1-\rho)^3}$$

and Table (1.3), it is found that equation (1.19) reduces to the equation,

$$AH'^3(1) + BH'^2(1) + C = 0 \qquad (1.20)$$

where $A = 8,$

$$B = 24 + \frac{6(4-2\rho)}{(1-\rho)}$$

and

$$C = (\ -1)\phi'^4(1) + \frac{24(2-\rho)^3}{(1-\rho)^2} + \frac{6\ (4-2\rho)}{(1-\rho)}\quad .$$

Upon taking further derivatives, each even numbered derivative seems to produce an identity and each odd numbered derivative seems to result in another equation in two more unknowns. This trend is expected to continue for all derivatives, but it has been verified only up to the eighth derivative.

These two methods produce two separate sets of equations that independently are not sufficient to solve for any of the higher order moments. Although it is straightforward to solve for the first order moment due to the symmetry that exists in these systems, the necessary independent equations needed to solve for the higher order moments are lacking. Together, the two sets of equations in the best case produce two equations in three unknowns. In the general case when the first method has (k+m-1) equations in 2(k+m-1) unknowns, the greatest number of independent equations the second method supplies is (k+m-1)/2 which is far short of the necessary (k+m-1) equations needed to solve for any of the unknowns. For this reason, the solution to higher order moments is still an open problem.

1.C. Evidence of Correlation in a Three-Node Slotted ALOHA Network

This section uses simulation to provide evidence that correlation exists in a three-node slotted ALOHA network. As a direct extension of the work done in [9] for a two-queue network, if the inequality shown to be true by simulation is always true, the throughput for a three-queue buffered ALOHA network can also be determined [2].

1.C.1. Model and Goals

The model used is a three-node slotted ALOHA broadcast system. Stations one and two have transmission probabilities f_1 and f_2 when they have a packet to transmit, respectively, and mean arrival rates a_1 and a_2, respectively. The third station has an infinite backlog and transmits with probability p in each slot. The state of the system is defined by the number of customers in station one and the number of customers in station two at the end of a slot. Using the methods discussed in subsection 1.A.2., a generating function form of the equilibrium equation, $\pi = \pi P$, where π is the joint steady-state density function and P is the transition probability matrix, is found to be

$$\begin{aligned} G(x,y) = A(x)A(y)\Big[&[G(x,y) - G(x,0) - G(0,y) + G(0,0)]B(x,y) \\ &+ [G(x,0) - G(0,0)]C(x) + [G(0,y) - G(0,0)]C(y) \\ &+ G(0,0)\Big] \end{aligned}$$

where

$$B(x,y) = (1 + \bar{p}f_1\bar{f}_2(x^{-1}-1) + \bar{p}f_2\bar{f}_1(y^{-1}-1)) \quad ,$$

$$C(x) = (1 + \bar{p}f_1(x^{-1}-1)) \quad ,$$

$$C(y) = (1 + \bar{p}f_2(y^{-1}-1)) \quad , \quad \text{and}$$

$$\bar{p} = (1-p),\ \bar{f}_1 = (1-f_1),\ \text{and}\ \bar{f}_2(1-f_2) \quad .$$

The A(x) and A(y) terms represent the generating function of the arrival process. In the model discussed here, they are represented by a Bernoulli arrival process with rates a_1 and a_2 so

$$A(x) = 1 + a_1(x-1), \text{ and}$$

$$A(y) = 1 + a_2(x-1)$$

are the generating functions of the arrival process.

The goal of this work is to show by simulation that the probability of being in the state where stations one and two are empty is less than the product of the probability that station one is empty and the probability that station two is empty. Written another way, this is equivalent to

$$G(0,0) \quad G(0,1)G(1,0) \tag{1.21}$$

The simulation determines estimates of $G(0,0)$, $G(0,1)$, and $G(1,0)$.

1.C.2. Simulation and Results

The simulation uses a set of conditions to determine the a_1, a_2, f_1, f_2, and p necessary to determine the unknowns in equation (1.21). The program used is written in the Fortran language and is in no way considered to be efficient. The program simulates arrivals and departures in an ALOHA system. The structure is as follows. It uses a main program to generate random variables and to call the first subroutine. The random numbers chosen between 0 and 1 are used to determine if an arrival or a departure took place later in the program. The first subroutine sets the parameters mentioned earlier such that they satisfy the following conditions,

$$p = (.1)i - 0.05 \qquad \text{for } i=1,10$$

$$f_1 = (.1)j - 0.05 \qquad \text{for } j=1,10$$

$$f_2 = (.1)k - 0.05 \qquad \text{for } k=1,10$$

$$a_1 = (.1)m - 0.05 \qquad \text{for } m=1,10$$

$$a_2 = (.1)n + 0.05 \qquad \text{for } n=1,10, \quad \text{and}$$

$$a_1 < f_1(1-f_2)(1-p)$$

$$a_2 < f_2(1-f_1)(1-p) \quad .$$

The last two conditions limit the region that the simulation is computed for by restricting the parameter to fall within a strongly stable region.

The second subroutine uses the passed parameters to stimulate an ALOHA system. A fifty-thousand slot run is made to compute the estimated number of times the system is in the states described earlier. An arrival occurs in a station if the parameter value is larger than an independent random number between 0 and 1 produced in the main program. A transmission occurs if the station is not empty, no other station is transmitting, and the parameter value for that station determined in the first subroutine is larger than the independent random number between 0 and 1 tabulated in the main program. The states of both stations are kept track of, and if either one, the other, or both stations, are empty, a flag is set corresponding to the condition present during that slot. The number of times a certain flag is set is kept track of and after the 50,000 slot run, the number of times a specific flag is set is divided by 50,000. These quantities are estimates of G(0,0), G(1,0), and G(0,1). A printout of the estimates of G(0,0) and the product G(0,1)G(1,0) is used to verify equation (1.21) for the parameter values used.

Over the total range of the parameters used, equation (1.21) held for the estimates. This does not say that equation (1.21) is true for all parameter values, but it does produce hope that a proof of equation (1.21) is possible.

Chapter II

Verification that Predictions are Valid for Control Strategies Using Simulations

2.A. Introduction

In this chapter, a time-slotted ALOHA broadcast channel is simulated using two different types of channel feedback to determine transmission probabilities. The simulations done are an extension of the work done in [3] where strategies based on stochastic approximations are presented to control transmission probabilities of stations in a collision access communication channel. The main change incorporated into the simulation is that the number of users is allowed to "slowly" vary.

In this section, goals and a review of the work done in [3] are presented. We describe in section B the simulations of the fixed user model with a nonzero ε, a parameter that in [3] is allowed to tend toward zero. The data from these simulations are then compared to the values given in [3]. In section C, the variable user models are presented. Simulations of these models are also done and the data from these simulations are reviewed.

2.A.1. Goals

The motivation of this work is to simulate the 0-1-e feedback and the acknowledgment based feedback strategies under the conditions that the parameter ε is strictly greater than zero and that the number of users is allowed to slowly vary in order to verify that the theory presented in [3] is capable of predicting simulation results. We want to see that under conditions where the theory cannot be applied directly to produce exact analytic results, as in both of the forementioned cases, it is still possible to predict the behavior of the control strategies.

2.A.2. Background Information

This subsection reviews work first presented in [3]. Only work relevant to the simulations done in this paper is presented. The models used throughout this paper consider N stations sharing a communication channel, where N is not assumed to be fixed unless stated so. The channel can be accessed only during specific time slots which result in a station j of the N possible stations transmitting a packet during time-slot k with probability $f_j(k)$. If station j transmits in slot k, $U_j(k)$ equals 1 and $U_j(k)$ equals zero otherwise. Station j receives channel feedback at the end of a slot. This feedback is represented by $Y_j(k)$. It is assumed that given $\underline{f}(k)$, the random vector given by $(\underline{U}(k), \underline{Y}(k))$ is conditionally independent of the past and that the conditional distribution of $(\underline{U}(k), \underline{Y}(k))$ given $\underline{f}(k)$ is not dependent on k. The transmission control strategies considered take on the form

$$f_j(k+1) = [f_j(k) + \varepsilon\, G_j(f_j(k), Y_j(k), U_j(k))]_0^{\Delta_j} \qquad (2.1)$$

where $\underline{\Delta} = (\Delta_1, \ldots, \Delta_N)$ is a vector with positive coordinates, $[x]_a^b$ denotes the projection, min {b, max {a,x}}, of x onto the interval [a,b], and the G_j's are bounded continuous functions.

By the given assumptions, there is an N-vector valued function $\underline{\overline{G}}$ defined by

$$\overline{G}_j(\underline{f}) = E\,[G_j(f_j(k),\, Y_j(k),\, U_j(k))\,|\,\underline{f}(k) = \underline{f}]. \tag{2.2}$$

By allowing a time transformation where $k\varepsilon = t$ for an integer k, it is possible to define a continuous-time random process f^{ε} by setting $f^{\varepsilon}(t) = f(k)$ where it is also required that the sample paths of f^{ε} be affine over intervals of the form $[k\varepsilon, (k+1)\varepsilon]$. This is equivalent to expanding time using the parameter t while ε tends to zero. Then using the theory of stochastic approximation, under general conditions, the random process $\underline{f}^{\varepsilon}$ converges weakly as ε tends to zero (essentially all that is needed is that the conditional distribution of $(\underline{U}(k), \underline{Y}(k))$ given $\underline{f}(k)$ should depend smoothly on $\underline{f}$, see [4, sections 5.3, 5.4, and 4.4]) to the solution of the following ordinary differential equation (O.D.E.) with boundary:

$$\begin{gathered}\dot{\underline{f}} = \overline{\underline{G}}(\underline{f}) + \underline{v} + \underline{u} \quad ; \ \underline{f}(0) \text{ given} \\ 0 \leqslant \underline{f}(t) \leqslant \underline{\Delta}\ ,\ \underline{v}(t) \leqslant 0,\ \underline{u}(t) \geqslant 0 \\ v_i(t)(\Delta_i - f_i(t)) = 0\ ,\ u_i(t)\, f_i(t) = 0\end{gathered} \tag{2.3}$$

where $\underline{u}$ and $\underline{v}$ are auxiliary functions to be determined together with $\underline{f}$. This convergence result is akin to the law of large numbers. A second convergence result, akin to the central limit theorem, pertains to the limiting distribution as ε tends to zero of the random process $\overline{\underline{f}}^{\varepsilon}$ defined by

$$\underline{f}^{\varepsilon}(t) = \frac{\underline{f}^{\varepsilon}(t) - \underline{f}^{*}}{\sqrt{\varepsilon}}$$

where $\underline{f}^*$ is a locally asymptotically stable equilibrium point of the ODE (2.3). To simplify the analysis, it is assumed that $0 < f_j^* < \Delta_j$ for each j. Then under general conditions (see [4, sections 5.6 and 4.7]), if $\tilde{f}^{\varepsilon}(0)$ converges weakly (as ε tends to zero) to a constant x(0), or if f^* is globally asymptotically stable and on initial transient period is ignored, $\tilde{f}^{\varepsilon}$ converges weakly to a Gaussian diffusion process described by the Itô stochastic equation

$$d\underline{x}(t) = F\underline{x}(t) + \Sigma^{1/2}\, d\underline{w}_t \ .$$

In this equation, $\underline{w}$ is a standard N-dimensional Wiener process,

$$F = \left.\frac{\partial \overline{G}}{\partial \underline{f}}\right|_{\underline{f}^*} \quad \text{and} \quad \Sigma = \operatorname{cov}(\xi \,|\, \underline{f}(k) = \underline{f}^*). \tag{2.4}$$

where $\xi_j = G_j(f_j(k), Y_j(k), U_j(k))$. The invariant measure for $(\underline{x}(t))$ is Gaussian with mean zero and covariance matrix Q satisfying

$$QF + FQ + \Sigma = \underline{0} \ . \tag{2.5}$$

If the diffusion approximation is sufficiently accurate, the covariance matrix of $\underline{f}(k)$ for large k should thus be $Q\varepsilon + o(\varepsilon)$.

Information on the asymptotic convergence rate of solutions to the ODE is given by the eigenvalues of F. The more negative the real parts of the eigenvalues, the quicker the convergence rate. The matrix Q gives a measure of steady state accuracy of the control strategy. As a specific case, $(a^TQa)\varepsilon$ gives the approximate steady state variance of $a \cdot f$, for any constant N vector a.

Strategies are considered such that

$$\overline{G}_j(\underline{f}) = (f_j)^r \, (\phi_N(\underline{f}) - \alpha A_j(f_j)) \quad \underline{o} \leqslant \underline{f} \leqslant \Delta \qquad (2.6)$$

where r and α are positive constants with $r \geqslant 1$ and $\alpha > o$, and the following conditions are satisfied for all j:

(condition 1) $\phi_N(\underline{f})$ is strictly decreasing in $\underline{f}$ and is continuously differentiable.

(condition 2.j) $A_j(0) = 0$, $A_j(u)$ is strictly increasing in u and is continuously differentiable, and $u^r A_j(u)$ has a finite right-hand derivative at $u = 0$.

These conditions ensure that $\overline{\underline{G}}$ is locally Lipschitz continuous so that the ODE (2.3) has a unique solution for any initial point. The restricted region of ODE (2.3) considered is

$$\Omega = \{\underline{f} \;:\; 0 < f_j \leqslant \Delta_j \quad \text{for each } j\} \;.$$

Since $\dot{f}_j \geqslant c f_j + o(f_j)$ for all j and for some negative c, the solution f(t) remains in Ω for all finite t. Therefore, the auxiliary variables in (2.3) are identically zero.

In [3], a Liapunov-style proof of global stability for the ODE (2.3) is presented. This proof, however, is omitted from this paper.

Two control strategies are presented which satisfy the conditions set forth earlier in this section. A detailed explanation of these strategies is given in [3], so all that is given here is a brief explanation of the two strategies. The first strategy considered uses the common 0-1-e feedback to adjust the transmission probabilities. For 0-1-e feedback, $Y_j(k)=Z(k)$ for each j, where $Z(k)$ is equal to 0, 1 or e depending on whether zero, one, or more than one station transmits in slot k. The transmission rule is of the form in Eq. (2.1) where

$$G_j(\underline{f}, y, \underline{u}) = (f_j)[c(y) - \alpha A_j(f_j)] \qquad (2.7)$$

for the vector $\underline{c}$ = (.418, 0, -.582) which is the vector suggested in [3] on the basis of a Poisson approximation. The arbitrary functions A_j also must satisfy condition 2.j for all j.

The second strategy considered uses the acknowledgment based transmission control to determine the transmission probabilities. This strategy relies only on the knowledge of whether or not each station's own transmissions are successful. The update rule given in [3] is (using Z(k) as above)

$$f_j(k+1) = [f_j(k) + \varepsilon f_j(k)\ c_j(f_j(k), Z(k), U_j(k))]_0^1$$

where $\Delta = 1 - e^{-1}$,

$$c_j(u,Z,U) = \begin{cases} (1-u)e - 1 - \alpha A_j(u) & Z=1, \quad U=1 \\ -1 - \alpha A_j(u) & Z=e, \quad U=1 \\ 0 & U=0 \end{cases}$$

and the functions A_j are arbitrary functions satisfying condition 2.j for all j. Both strategies given here are simulated under the conditions given in section (2.A.1.), and the results of these simulations are given in the final two sections.

The results of the simulations are compared to predictions determined as follows. To simplify the simulations, all of the functions A_i are the same for all i. This results in the globally stable equilibrium point $\underline{f}^*$ of the ODE (2.3) having the form $\underline{f}^* = (u, \dots , u)$, where u is easily computed. The F and Σ matrices defined in (2.4) have the form

$$F = aI + bW, \quad \Sigma = cI + dW \tag{2.8}$$

where a, b, c, d are readily computed and W is the matrix of all ones, and so the scaled covariance matrix Q determined by (2.5) also has the form $Q = rI + sW$, where r and s can be readily expressed in terms of a, b, c, and d. In general, a and b are less than or equal to zero. F has eigenvalue a+b with multiplicity one (corresponding to the eigenvector of all ones), and the eigenvalue a with multiplicity (n-1). The smallest magnitude eigenvalue of F is used as an indicator of how quickly the individual transmission probabilities should fall into line, or of the relaxation time seen in the system. The predictions given in [3] for the "relaxation time" as well as the expected throughput and the equilibrium value of $\underline{f}^*$ are compared to simulation values in the final two sections of this paper.

The control of the sum of transmission probabilities is also used to determine how well simulation values conform to predictions. The sum is written as $\underline{e} \cdot \underline{f}$ where e is the all ones row vector. An estimate for the steady state variance of each f_j is found by looking at the diagonal entries of the matrix Q and multiplying this by [illegible]. $e^T Q e$ approximates 1/[illegible] times the steady state variance of $\underline{e} \cdot \underline{f}$. Also, the magnitude of the eigenvalue of the matrix F for eigenvector e gives a speed of convergence measure for $\underline{e}\cdot\underline{f}$. These measures are used to determine if the simulated models follow the predicted values found using the theory given.

2.B. Simulation of Nonzero ε, Fixed N

2.B.1. Models and Simulation

The models used in this section are specific examples of the models given in the previous section. In this section, N is fixed; however, ε is not allowed to tend to zero as is the case in the previous section. The update rules are the same as those given in the previous section with $A_j(u)=u$ for all j. The choice of the α and ε values is discussed in the following subsections along with the simulation. These simulation results are then compared to the predictions made in [3].

A Fortran program is used to simulate the strategies. The simulations use a uniform random number generator to determine which, if any, stations transmit. Then, transmission probabilities are updated according to the strategy used and the feedback received. The program prints out the throughput and the transmission probabilities for each station at predetermined intervals throughout the simulation. An estimate for the average transmission probability is determined over the total number of slots excluding the first thousand slots in order to reduce transient effects. Various initial conditions are used for the simulations. The initial transmission probabilities are chosen randomly for the most part. In this way, predictions concerning the transient period, such as the relaxation times, can be checked. An estimate for the variance of the individual transmission probabilities and of the sum of transmission probabilities using the same sample space as that used to determine the mean is also evaluated. All of these estimates are printed out at the end of a simulation run and, for some of the runs, discussed in the next subsection.

2.B.2. Results of the N Fixed Case

The predictions, from [3] and the estimates from the simulations are compared in this subsection. This comparison is used to determine if the strategies perform as expected, and hence provide proof that the simulations bear out the theory. All simulations are run over 4000 to 5000 slots, the only exceptions being in cases noted in this subsection where the number of slots was allowed to be 10,000 and 30,000 slots.

The simulations for the 0-1-e strategy all exhibit the fact that once two or more stations' transmission probabilities become approximately equal (i.e., fall into line), the probabilities remain equal throughout the remainder of the simulation. The transmission probabilities are adjusted by the same update rules since the same feedback is used by each of the stations. This is unlike the acknowledgment based strategy where each of the stations updates its own transmission probabilities using its own feedback determined by whether or not it transmits in a slot.

Table (2.1) shows the throughout and Table (2.2) shows the estimated mean transmission probabilities for various values of ε and N, and a

Table (2.1) Throughput results for the 0-1-e strategy for $\varepsilon=.05$ and for various N and α, given in successful transmissions per slot

		N=2	N=6	N=10
$\alpha=.088$	expected	.49	.40	----
	simulated	.47	.39	----
$\alpha=.25$	expected	.46	.40	----
	simulated	.47	.39	----
$\alpha=.707$	expected	.39	.39	.38
	simulated	.39	.40	.38
$\alpha=1.0$	expected	.35	.38	.38
	simulated	.35	.37	.37

Table (2.2) Transmission probabilities for the 0-1-e strategy for $\varepsilon=.05$ and for various N and α

		N=2	N=6	N=10
$\alpha=.088$	expected	.42	.16	----
	simulated	.41	.14	----
$\alpha=.25$	expected	.37	.15	----
	simulated	.36	.15	----
$\alpha=.707$	expected	.26	.13	.08
	simulated	.26	.13	.08
$\alpha=1.0$	expected	.22	.12	.08
	simulated	.22	.11	.08

value of .05 for ε. These values are compared to predicted values in the same tables. In all cases considered, the simulated values are near the predicted values. The error between the simulated and the predicted values is given by the steady state accuracy. This steady state accuracy is measured by the variance about the mean of the transmission probabilities. This is discussed in more detail later in this subsection. The lack of results for the N=10 cases where α=.088 and α=.25 is due to the convergence problems present when a small α is chosen. As α gets smaller, the smallest magnitude eigenvalue decreases in magnitude for a constant N. This results in a longer relaxation time which requires a longer period of time prior to the recording of data due to a longer transient period. This in turn requires more memory and CPU time from the computer which soon becomes expensive. For this reason, and the fact that the simulations done produce adequate proof that the theory predicts the simulation results for these cases, the N=10 runs are omitted.

We now consider the effect of changing the values of $\underline{\alpha}$ and of $\underline{\varepsilon}$ on the steady state variance and the relaxation time. As $\underline{\alpha}$ and $\underline{\varepsilon}$ are increased it should take less time to converge to the predicted transmission probabilities. However, this causes the variance of the transmission probabilities to increase which is equivalent to saying the steady state accuracy is degraded. On the other hand, an α and an ε that are smaller in magnitude result in a longer relaxation time which in turn causes a longer transient period prior to convergence as in the N=10 case discussed earlier in this subsection. The effects of varying the $\underline{\alpha}$ and $\underline{\varepsilon}$ values are much more dramatic in the acknowledgment based strategy where the transmission probabilities never actually become or stay equal over any length of time, resulting in larger variances and in larger relaxation times.

This leads to the results presented in Tables (2.3), (2.4), and (2.5). Table (2.3) produces a comparison of the predicted variances and the estimated variances found in the simulations. Again, in most cases the estimates from the simulations compare well with the predicted values. Table (2.4) presents predicted relaxation times found using the theory presented in [3]. Note that for N=10, the relaxation times for α=.088 are approximately 2400 slots. For this reason, the α=.088 and N=10 case is omitted. The N=6 and α=.088 case also has a long relaxation time; however, a 10,000 slot run is done for this case. Only one 10,000 slot run is performed due to the cost of producing such simulations. It is also noted that for N=2, the relaxation times all lie below 600 slots. In the runs done for N=2, for the most part, the time needed for the transmission probabilities to converge to a common value is well within the 600 slot relaxation times given. The relaxation times are also dependent upon the initial transmission probabilities used by the stations. However, no analysis is done on this transient period to determine the effects of various initial conditions. Rather, the only real interest in this transient period concerns ensuring that the transmission probabilities do converge under random initial conditions to the predicted values, which in all cases they do. In Table (2.5), the estimated variances found for various β are given with N=6 and α=1.0. Using the theory given earlier, it is known that the variances and the relaxation times

Table (2.3) Steady state variance of individual transmission probabilities for the 0-1-e strategy where ε=.05 and for various N and α

		N=2	N=6	N=10
α=.088	expected	.001	.0002	----
	simulated	.001	.0005	----
α=.25	expected	.001	.0002	----
	simulated	.001	.0002	----
α=.707	expected	.0003	.0001	.0001
	simulated	.0003	.0001	.0001
α=1.0	expected	.0002	.0001	.0001
	simulated	.0002	.0001	.0001

Table (2.4) Relaxation times for the 0-1-e strategy where ε=.05 and for various N and α

	N=2	N=6	N=10
α=.088	540	1500	2400
α=.25	220	550	880
α=.707	110	230	340
α=1.0	90	170	260

Table (2.5) Variance for various ε and N=6, α=1.0

	ε=.5	ε=.05	ε=.005
expected	.001	.0001	.00001
simulated	.001	.0001	.00001

for that matter, are directly affected by the value chosen for ε. The choice of $\varepsilon=0.05$ for the runs already discussed is a value originally chosen after some experimentation with various values of ε. This value proved to be sufficient for the simulations done, but by no means is it assumed to be the best choice of ε possible.

The final results for the 0-1-e strategy are summed up in Tables (2.6) and (2.7). In Table (2.6), the predicted and estimated variance of the sum of the transmission probabilities is given. This measure of steady state accuracy provides a means to see how well as a whole the transmission probabilities have converged to the predicted mean sum. Again, the results appear favorable. Table (2.7) provides the relaxation times for the sum of the transmission probabilities. This presents an idea as to how quickly the sum converges for the parameter values discussed.

This leads to the discussion of results for the simulation of the Ack strategy. Tables (2.8), (2.9), (2.10), (2.11), (2.12), and (2.13) present the information analogous to that presented in the previous tables for the 0-1-e strategy. In Table (2.8), the predicted throughput and estimated throughput are given. Again, the estimates follow the predictions for each of the cases. Table (2.9) presents the predicted mean transmission probability and the simulated estimated mean with a range of values the individual means took at the conclusion of the simulation. For the Ack strategy, as discussed earlier, the individual stations have transmission probabilities which do not ultimately stay equal. For this reason, the estimated means of the individual stations take on a range of values. The theory presented

Table (2.6) Variance of sum for 0-1-e strategy for ε=.05 and various N and α

		N=2	N=6	N=10
α=.088	expected	.005	.008	----
	simulated	.004	.008	----
α=.25	expected	.003	.007	----
	simulated	.005	.007	----
α=.707	expected	.001	.004	.006
	simulated	.001	.005	.006
α=1.0	expected	.001	.004	.005
	simulated	.001	.003	.005

Table (2.7) Relaxation times of the sum of transmission probabilities for ε=.05 and various N and α

	N=2	N=6	N=10
α=.088	80	260	430
α=.25	80	210	340
α=.707	70	140	220
α=1.0	60	120	180

Table (2.8) Throughput for Ack strategy for ε=.05 and various values of N and α, given in successful transmissions per slot

		N=2	N=6	N=10
α=.088	expected	.47	.40	----
	simulated	.50	.44	----
α=.25	expected	.46	.40	----
	simulated	.47	.40	----
α=.707	expected	.44	.40	.39
	simulated	.44	.39	.39
α=1.0	expected	.43	.38	.38
	simulated	.40	.38	.38

Table (2.9) Transmission probabilities for Ack strategy for ε=.05 and various values of N and α (with an average of estimates and a range of estimates for the simulated case)

		N=2	N=6	N=10
α=.088	expected	.38	.15	----
	simulated	.35(.21-.50)	.13(.03-.26)	----
α=.25	expected	.37	.15	----
	simulated	.36(.33-.39)	.14(.06-.20)	----
α=.707	expected	.33	.14	.09
	simulated	.32(.30-.33)	.13(.08-.19)	.08(.008-.17)
α=1.0	expected	.31	.14	.09
	simulated	.29(.25-.33)	.12(.07-.17)	.08(.03- .16)

Table (2.10) Steady state variance of individual transmission probabilities for the Ack strategy where ε=.05 and for various N and α. (Note that the simulated variance is an average of all the estimated variance and a range of values is also given.)

		N=2	N=6	N=10
α=.088	expected	.02	.04	----
	simulated	.005(.004-.005)	.008(.00008-.02)	----
α=.25	expected	.006	.015	----
	simulated	.003(.002-.004)	.001(.0002-.003)	----
α=.707	expected	.003	.005	.004
	simulated	.003(.002-.003)	.001(.0005-.003)	.001(.000-.005)
α=1.0	expected	.002	.004	.003
	simulated	.002(.002-.002)	.002(.0002-.004)	.0006(.00005-.001)

Table (2.11) Relaxation times of individual transmission probabilities for the Ack strategy where ε=.05 and for various N and α

	N=2	N=6	N=10
α=.088	1500	9900	25000
α=.25	600	3600	9200
α=.707	270	1400	3500
α=1.0	210	1100	2600

Table (2.12) Variance of sum for Ack strategy for $\varepsilon=.05$ and various N and α

		N=2	N=6	N=10
$\alpha=.088$	expected	.006	.007	----
	simulated	.004	.040	----
$\alpha=.25$	expected	.006	.006	----
	simulated	.006	.006	----
$\alpha=.707$	expected	.005	.006	.004
	simulated	.004	.005	.005
$\alpha=1.0$	expected	.004	.005	.004
	simulated	.004	.005	.004

Table (2.13) Relaxation times of the sum of transmission probabilities for $\varepsilon=.05$ and various N and α

	N=2	N=6	N=10
$\alpha=.088$	80	680	1900
$\alpha=.25$	80	620	1700
$\alpha=.707$	70	510	1300
$\alpha=1.0$	70	470	1200

earlier predicts this result as the data in Tables (2.10) and (2.11) illustrate. Table (2.10) contains the predicted variances and the simulated range of variances for the various cases. It is seen that these variances are quite large compared to the variances of the 0-1-e strategy. Hence, it is expected that the various transmission probabilities vary more about the mean than in the 0-1-e strategy. Table (2.11) presents the predicted relaxation times again found from the theory discussed earlier. It is noted that these times are much greater than the times for the 0-1-e strategy. Hence, it is expected that a larger transient period also influences the results already presented. Since all the runs already discussed are over relatively short simulation times, 10,000 slots or less, a run of 30,000 slots is done to minimize transient effects, although the results given thus far for the simulations done do follow the predictions from the theory consistently. The run of 30,000 slots for N=10, α=.707, and ε=.05 is discussed later in this subsection.

Tables (2.12) and (2.13) contain the analogous data to that given for the variances of the sum and relaxation times of the sum for the 0-1-e strategy. Again, the simulation results are consistent with the predictions. It is worth noting that these results imply that the sum converges rather quickly, compared to the individual station's relaxation times, to a certain value; however, it is known from the given results for the individual stations that the individual transmission probabilities vary to a much larger extent. This implies that a judgment as to how well a simulation is following predictions cannot be made by looking only at the throughput and results concerning the sum of transmission probabilities. So, to alleviate any problems over ambiguity of results and over terribly long transient periods, a 30,000 slot simulation is done.

The parameters used for this run are ε=.05, α=.707, and N=10. The choice of 30,000 slots is due to limitations in the computer system employed to do this simulation. This allows ample time for recoveries by all the stations since the predicted relaxation time is 3500 slots. Table (2.14) contains some of the data found for this simulation and compares these to the predicted values. Again, it is seen that the simulation results are consistent with the predictions. This, however, says very little about whether individual stations recover from cases where a station's transmission probability falls well above or well below the expected value. To say something about this phenomenon, a definition of recovery is needed. A rough definition of recovery used in this paper is that a recovery occurs when a station's transmission probability returns from a value outside of plus or minus one standard deviation about the mean to a value within one-half the standard deviation about the mean. This definition is used here only as a measure of recovery for this simulation. Other possible definitions of recovery exist; however, this is the definition adopted here.

Table (2.15) presents a short excerpt from the data obtained from the simulation. These are just five examples of recoveries from a total of thirty-six recoveries which occurred using the definition discussed earlier. Again, the predictions hold true first in allowing for stations to fall outside of the recovery region and second in

Table (2.14) Results for the N=10, 30,000 slot simulation

	Expected	Simulated
Throughput	.39	.39
Transmission Probabilities	.09	.08 (.04-.13)
Variance	.004	.002 (.0003-.003)
Relaxation time	3500	----
Variance of sum	.004	.005
Relaxation time of sum	1300	----

Table (2.15) Examples of recovery times for various stations under the definition of recovery given in the text

	Transmission Probabilities		
Slot Number	Station s	Station 4	Station 10
6000	.269		
6500	.158		.024
7000	.118		.022
7500			.035
8000			.032
8500			.051
9000	.185	.023	.098
9500	.177	.020	
10000	.236	.023	
10500	.165	.028	.205
11000	.214	.027	.266
11500	.206	.036	.122
12000	.189	.054	
12500	.112	.105	

predicting relaxation times that most of the recoveries fell within. The average recovery time over the thirty-six recoveries is found to be 3200 slots for the simulation done. The final set of predictions concerns those made for the variance of the sum and the relaxation time of the sum. These results are also presented in Table (2.14). Again, the predictions are consistent with the relaxation time found for the simulation falling within the predicted value.

All these results supply evidence that the theory developed in [3] and presented in a condensed form in an earlier subsection provides consistent predictions as to the behavior of the control strategies with nonzero ε. Although little is said about how useful these strategies are in reality, this has nothing to do with the main result that it is possible to predict the performance of the strategies with the given theory. The simulations provided some proof of this. As a further variation and test of the power of the theory, predictions concerning the performance of a varying N case are compared to simulation results as done in this subsection.

2.C. N Allowed to Slowly Vary

2.C.1. Models and Simulation

The model discussed in this subsection differs from the model presented in the previous section only in that this model allows the number of stations, N, to vary in the simulation. The control strategies and the basic programming format are the same as those given in subsection 2.B.1.

Two different methods are employed to vary N. The first is a deterministic strategy where the number of stations is given by a sine function as follows:

$$N(s) = A + B \text{ Integer } [.1 + \sin(Cs)] \tag{2.9}$$

where N(s) is the number of stations used during slot s; A and B are constants determining the range of N(s); C is a constant in radians per slot denoting the frequency of change, and Integer [ ] takes the integer part of the expression in the brackets. This defines the deterministic system. Although the number of stations and the points in time where the number of stations change are known, the choice of users to be used for a particular length of time is made randomly. For example, consider the case where there are six particular stations out of a possible 15 stations using the channel for the next 30 slots, and after that the number of stations is increased to seven for the next 20 slots. The seven stations need not include all the six previous stations since at each change it is equally probable that each of the individual stations will be used in the next set of time slots.

The second method employed chooses both the number of stations and the number of slots these stations are "on" randomly from a set of possible stations and slot times. This method is called the random system. The choice of the number of stations used is dependent on a uniformly distributed random variable which takes on values from 0 to 1. The function used to determine N is given as follows:

$$N = K + \text{Integer } [L + R] \tag{2.10}$$

where K and L denote constants determining the range of N. R is a uniformly distributed random number as discussed above. Again the choice of which N stations that are "on" for a certain period is done randomly from the set of total possible stations. The period time, or the amount of time prior to changing the number of stations again, is determined by another uniformly distributed random variable which takes on values from 0 to 1. This function is given as follows:

$$s = D + \text{Integer}\,[E\ R] \tag{2.11}$$

where D and E determine the range of s, and R is the same as defined above.

Another choice made for the simulations is the choice of initial conditions for each station. Each station transmits with probability 1 for the 0-1-e strategy and $(1-e^{-1})$ for the Ack strategy when it initially turns on. However, two different methods are used when the stations are turned on in the simulation after having already been turned on and then off. The transmission probability is set to one for the 0-1-e strategy or $2\,(1-e^{-1})$ for the acknowledgment based strategy in the first method. The second method allows each station to save its final transmission probability prior to being turned off, allowing it to be used again when that station is turned on again. These methods are used only to gain insight into how to choose initial conditions under these circumstances. No analysis is done to determine which method is superior.

When the number of stations change, the throughout up to that point is printed out. The number of stations and the time slot are also printed out. The throughput given from the simulation is then compared to the predicted throughput from the theory in [3] to determine how well the simulation is predicted. Throughput may not be the most informative measure of performance since the individual transmission probabilities are not compared to their predicted values. The throughput is a simple measure that tests the predictions for the case that N is slowly varied.

2.C.2. Comparison of the Predicted Throughputs and of the Simulated Throughputs

The simulated throughputs are summarized for both strategies in Table (2.16) for the specified conditions. These throughputs are found using the values given for the parameters in equations (2.9), (2.10), and (2.11) as follows:

$A = 10$ $C = 2/300$ $E = 100$ $L = 11$

$B = 5$ $D = 20$ $U = 5$

and using 600 slot runs.

The theory yields, for given ε, α, and average N values, a relaxation time such that if N is slowly varying in a time scale on the order of the relaxation time then the transmission probabilities are controlled to within a predicted tolerance. For the systems discussed, an average N of 10 is used as the N needed to predict the relaxation times. In Table (2.17), the relaxation times that cannot be exceeded if the theory's predictions are to hold are presented. Also, in Table (2.17) there are the predicted throughputs for the cases that are simulated.

Table (2.16) Throughputs found for the variable N simulations where $\alpha=.25$ and the runs covered 6000 slots.

Note: $^{\bullet}$ has reinitialized to one policy on top
$/_{\bullet}$ has reinitialized to old value policy on bottom

Random

	Ack	0-1-e
$\varepsilon=.05$	$^{.28}/_{.34}$	$^{.26}/_{.25}$
$\varepsilon=.5$	$^{.35}/_{.39}$	$^{.36}/_{.36}$

Deterministic

	Ack	0-1-e
$\varepsilon=.05$	$^{.30}/_{.35}$	$^{.33}/_{.28}$
$\varepsilon=.5$	$^{.39}/_{.34}$	$^{.37}/_{.36}$

Table (2.17) Predicted throughputs, relaxation times, and relaxation times of the sum for N=10, $\alpha=.25$

Ack	Throughput	Relaxation Time	Relaxation Time at Sum
$\varepsilon=.0$	.39	9200	1700
$\varepsilon=.5$	.39	920	1700
0-1-e			
$\varepsilon=.05$	.39	870	340
$\varepsilon=.5$	.39	90	30

The random system is designed to have an average period of 70 slots prior to a change in the number of stations. The deterministic system is designed so that the number of slots it takes to go from 5 to 15 stations is 150. Hence, the models of varying N which we used in our simulation have N roughly constant over time periods on the order of 70 slots for the random system and 100 slots for the deterministic system. These time periods exceed the theoretical relaxation times in only one of the four different cases, namely the case where $\varepsilon=.5$ and $\alpha=.25$ for the 0-1-e. Otherwise, the N is varying faster than the theory allows, so under these cases the series need not apply to predicting the throughput. A solution to this is to slow down the rate of variation of N allowing the resulting relaxation times to be larger than what is required for the theory.

In the cases that the theory does apply, it is seen that the results are fairly accurate in that the simulated throughputs approximate the predicted throughputs. When the theory does not apply, as in the other three cases, the strategies appear to still achieve good performance (i.e., high throughput). Here, a more "global" theory is needed to predict performance when the local theory described in [3] does not apply.

It is noted that any comparison of the two policies as to how to set the transmission probabilities when a station turns on after already being on once produces no useful results. Neither policy appears to be better than the other using the data in Table (2.16) as the measure. Hence, no conclusions are raised about this point.

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